On experimentally uncertainty quantification and machine learning in dynamics of composite structures

K. Sepahvand

Department of Mechanical Engineering
Technical University of Munich (TUM)
Germany

Keynote in
UNCECOMP 2019
24-26 June 2019, Crete, Greece
Survey

1. Introduction

2. Identification of parameter distribution

3. Spectral based method

4. Deep learning based method

5. Case study
Introduction

1-Forward vs. inverse problem

Forward problem

\[ U = M(P) \]  \hspace{1cm} (1)

- \( U \): system response
- \( M \): operator (model)
- \( P \): parameters

Inverse problem (parameter identification)

\[ P = M^{-1}(U) \]  \hspace{1cm} (2)

Challenges:

- \( M^{-1} \) may not exist (e.g. nonlinearity)
- ill-posed\(^1\)
- data involves some level of uncertainties

\(^1\)_well-posed: a solution exists and is unique
Introduction

2-Deterministic vs. stochastic parameter identification

▶ **Deterministic**: unique prediction of parameters, output may be dramatically different from measurements

▶ **Stochastic**: parameters are described by random variables rather than single values, yields a manifold of equally likely measurements

\[ y(x) = a_0 + a_1 x + a_2 x^2 \]
Introduction

3-Simulation with uncertain parameters

\[ X(\xi) \rightarrow \text{Simulation model} \rightarrow Y(\xi) \]
Introduction
3-Simulation with uncertain parameters

$X(\xi)$  
Simulation model  
$Y(\xi)$

Experimental data on $X$
Introduction

3-Simulation with uncertain parameters

\[ X(\xi) \quad \text{Simulation model} \quad Y(\xi) \]

True distribution of \( X \)  
Experimental data on \( X \)

Experimentally UQ and ML in composites
Introduction
3-Simulation with uncertain parameters

\[ X(\xi) \quad \text{Simulation model} \quad Y(\xi) \]

- Prior PDF
- Posterior PDF

True distribution of \( X \)
Experimental data on \( X \)

\[ \text{Experimental UQ and ML in composites} \]
Introduction

4-Discretization of random parameters

FEM model of structure
Discretized random parameter: FEM adapted discretization
Semi-discretized random parameter: random dim. independent of FEM mesh
Introduction
4-Discretization of random parameters

Lumped random parameter: randomization between samples
Elastic modulus as:

1) Random variable

\[ E(\xi) \]

2) Random field

\[ E(x, \xi) \]

\[ E_1(x_1, \xi_1) \quad E_2(x_2, \xi_2) \quad \ldots \quad E_n(x_n, \xi_n) \]

\[ c(x_i, x_j) = e^{-|x_j - x_i|/l_c} \]
Model with uncertain parameters:

\[ M(\xi)\ddot{u}(\cdot, \xi) + C(\xi)\dot{u}(\cdot, \xi) + K(\xi)u(\cdot, \xi) = F \]  

(3)

\( \xi \): vector of random variables (random dimensions)
Model with uncertain parameters:

\[ M(\xi)\ddot{u}(\cdot, \xi) + C(\xi)\dot{u}(\cdot, \xi) + K(\xi)u(\cdot, \xi) = F \]  

(3)

\( \xi \): vector of random variables (random dimensions)

Two cases may happen:

I. **Given observed data on the parameters**
   - **Goal**: identification of statistical properties, e.g. PDF, of parameters

II. **Given observed data on the system responses**
   - **Goal**: identification of statistical properties of parameters via an inverse problem
The uncertain $\mathcal{P}$ is distributed by $F_p(\Theta)$ having parameters $\Theta$:

1. Experimental data on $\mathcal{P}$ is available
2. It can be characterized as random variable $\mathcal{P}(\xi(\Theta))$
3. $\mathcal{P}$ has a unknown probability function $F_p(\Theta)$

**Goal:** identification of parameter set $\Theta = \{\theta_1, \theta_2, \ldots, \theta_m\}$

\[
\hat{\Theta} \sim f(\text{date}; \text{parameter}) = f(D; \Theta)
\]

$f$: probability density function (PDF)
Identification of parameter distribution
Frequentist vs. Bayesian identification

Frequentist method
- unknown fixed parameters $\Theta$
- Assuming initial values for parameters
- Estimates $\Theta$ with some confidence
  $$\hat{\Theta} = \arg \max_{\Theta} \mathcal{L}(D \mid \Theta)$$

Bayesian method
- Uncertainty in unknown parameters
- Assuming prior probability for parameters
- Estimates $\Theta$ as random variables
  $$f(\Theta \mid D) \propto \mathcal{L}(D \mid \Theta) f(\Theta)$$
  - Posterior
  - likelihood
  - Prior

© K. Sepahvand
Experimentally UQ and ML in composites
10
Identification of parameter distribution
Frequentist vs. Bayesian identification

- Bayesian approaches likelihood as the number of samples increases
- Limited data available: use Bayesian method!
Let $P(\xi)$ be approximated using a spectral expansion, e.g. generalized Polynomial Chaos (gPC) as

$$P(\xi) = \sum_{i=0}^{N} a_i \Psi_i(\xi) = a^T \Psi$$

(4)

random basis $\Psi_i(\xi)$ has the orthogonality relation of

$$\langle \Psi_i, \Psi_j \rangle = E[\Psi_i, \Psi_j] = E[\Psi_i^2] \delta_{ij} = h_i^2 \delta_{ij}$$

(5)

with $h_i^2$ as the norm of polynomials.

Deterministic coefficients $a = \{a_0, a_1, \ldots, a_N\}^T$, calculate as

$$a_i = \frac{\langle P(\xi), \Psi_i(\xi) \rangle}{\langle \Psi_i^2(\xi) \rangle}$$

(6)
Spectral based identification

Statistical moments

The $k^{th}$ order statistical moment of $\mathcal{P}$ from gPC

$$m_k = \mathbb{E} [\mathcal{P}^k] = \int_{\Omega} \left[ \sum_{i=0}^{N} a_i \Psi_i(\xi) \right]^k f(\xi) \, d\xi \quad (7)$$

e.g. 1$^{st}$ and 2$^{nd}$ moments:

$$\mathbb{E}[\mathcal{P}] = m_1 = \sum_{i_1=1}^{N} a_{i_1} \langle \Psi_0, \Psi_{i_1} \rangle = a_0$$

$$\mathbb{E}[\mathcal{P}^2] = m_2 = \sum_{i_1=1}^{N} \sum_{i_2=1}^{N} a_{i_1} a_{i_2} \langle \Psi_{i_1}, \Psi_{i_2} \rangle$$

the $k^{th}$ order central statistical moment ($m_0 = 1$)

$$\mu_k(a) = \mathbb{E} \left[ (\mathcal{P} - \mathbb{E}[\mathcal{P}])^k \right] = \sum_{i=0}^{k} \binom{k}{i} (-1)^{k-i} m_i m_{k-i}^{k-i}, \quad k = 2, 3, \ldots \quad (8)$$
The $k^{th}$ order statistical moment of $\mathcal{P}$ from gPC

$$m_k = E[\mathcal{P}^k] = \int_{\Omega} \left[ \sum_{i=0}^{N} a_i \Psi_i(\xi) \right]^k f(\xi) \, d\xi$$

\((7)\)

e.g. 1$^{st}$ and 2$^{nd}$ moments:

$$E[\mathcal{P}] = m_1 = \sum_{i_1=1}^{N} a_{i_1} \langle \Psi_0, \Psi_{i_1} \rangle = a_0$$

$$E[\mathcal{P}^2] = m_2 = \sum_{i_1=1}^{N} \sum_{i_2=1}^{N} a_{i_1} a_{i_2} \langle \Psi_{i_1}, \Psi_{i_2} \rangle$$

the $k^{th}$ order central statistical moment ($m_0 = 1$)

$$\mu_k(a) = E[(\mathcal{P} - E[\mathcal{P}])^k] = \sum_{i=0}^{k} \binom{k}{i} (-1)^{k-i} m_i m_1^{k-i}, \quad k = 2, 3, \ldots \ \ (8)$$

identification of $\mathcal{P} \equiv$ identification of $a_i$
Spectral based identification
Identification of $a_i$ from data

Let $\hat{p} = \{p_1, p_2, \ldots, p_M\}$ be $M$ sample data available on $\mathcal{P}$. The $k^{th}$ order statistical moments of data:

$$\hat{\mu}_k = \frac{1}{M} \sum_{n=1}^{M} (p_n - \hat{\mu})^k$$

$\hat{\mu}$: mean value

**Identification of $a = \{a_0, a_1, \ldots, a_N\}$**

$$\arg\min_{\alpha} \|L_2(\alpha)\| = \sum_{i=1}^{k} w_i \left( \mu_i(\alpha) - \hat{\mu}_i \right)^2$$

This is all a deterministic process.
Spectral based identification
Algorithm

Data on responses

Stat. moments of responses

gPC of responses
$U(\xi) = \sum_{i=0}^{N} u_i \Psi_i(\xi)$

Initialization of parameter’s gPC, $P(\xi) = \sum_{i=0}^{N} a_i \Psi_i(\xi)$

Simulation Model

$u_{i}^{(k+1)} - u_{i}^{(k)} \leq \varepsilon$?

Update $a_i^{(k)}$

No

PDF of parameters

$\Psi_i(\xi)$

Yes

$a_i$

Experimentally UQ and ML in composites

© K. Sepahvand

15
Deep Learning based method
PDF identification with limited data

Artificial Intelligence
Techniques enabling computers to mimic human activities

Machine Learning
Ability to learn without explicit being simulated

Deep Learning
Learn underlying features of data using neural networks

Steps
▶ Data preprocessing
▶ Network architecture
▶ Network training
▶ Simulation
▶ Post-processing

© K. Sepahvand
Experimentally UQ and ML in composites
Deep Learning based method
PDF identification with limited data

Data is usually divided into three subsets:

1. **Training set**: for updating the network weights and biases
2. **Validation set**: to monitored the error during the training process
3. **Test set**: to compare the accuracy for generating different models

\[ f(x) = p_1 + p_2 x + p_3 \frac{\sin(x)}{x} \]

![Graph showing a comparison between different data sets and models]
Case study
Identification of uncertain material parameters

Experimental modal analysis on identical sample plates (190 × 100 × 10 mm³)

© K. Sepahvand
Experimentally UQ and ML in composites 18
Case study
Damping as uncertain parameter

DL based identification of the first mode damping

![Graph showing DL based identification of the first mode damping.](image)
Case Study
Reduced order of the data

First 9 identified natural frequencies [Hz] for identical plates

original dimensions $n = 9$  
reduced dimensions $k = 3$

- $f_1$
- $f_2$
- $f_3$
- $f_4$
- $f_5$
- $f_6$
- $f_7$
- $f_8$
- $f_9$

○ True value  + Reduced dim.
Case study
Identification of uncertain material parameters

Distribution of the elastic parameters

- $E_{11}$ [Gpa]
- $E_{22}$ [Gpa]
- $G_{12}$ [Gpa]
- $\nu_{12}$ [-]

©K. Sepahvand
Experimentally UQ and ML in composites
Conclusion

1. Identification of uncertain parameters from data, direct or inverse problems
2. Spectral-based methods, identification of deterministic coefficients instead of random parameters
3. Statistical moments, identification of coefficients by LS
4. Simulation model, as black-box, no modification required
5. Reduced dimension in random domain
6. Machine Learning based identification (ongoing work)
Conclusion

1. Identification of uncertain parameters from data, direct or inverse problems

2. Spectral-based methods, identification of deterministic coefficients instead of random parameters

3. Statistical moments, identification of coefficients by LS

4. Simulation model, as black-box, no modification required

5. Reduced dimension in random domain

6. Machine Learning based identification (ongoing work)

Thank you for your attention!

Email: k.sepahvand@tum.de
Introduction
Probabilistic description of random quantities

- Random variables: $X(\xi) : \Omega \mapsto \mathbb{R}^d$, $d = 1, 2, 3$
  - essential Info: probability distribution
Introduction
Probabilistic description of random quantities

- Random variables: \( X(\xi) : \Omega \mapsto \mathbb{R}^d \), \( d = 1, 2, 3 \)
  - essential Info: probability distribution

- Random process: temporal random distributed, \( X(t, \xi) : T \times \Omega \mapsto \mathbb{R}^d \)
  - essential Info: correlation function
    \[ C(t_1, t_2) = \text{corr}(X(t_1, \xi), X(t_2, \xi)) \]
Introduction
Probabilistic description of random quantities

► Random variables: \( X(\xi) : \Omega \mapsto \mathbb{R}^d, \quad d = 1, 2, 3 \)
  o essential Info: probability distribution

► Random process: temporal random distributed, \( X(t, \xi) : T \times \Omega \mapsto \mathbb{R}^d \)
  o essential Info: correlation function
    \[ C(t_1, t_2) = \text{corr}(X(t_1, \xi), X(t_2, \xi)) \]

► Random field: spatial random distributed, \( X(x, \xi) : D \times \Omega \mapsto \mathbb{R}^d \)
  o essential Info: correlation function
    \[ C(x_1, x_2) = \text{corr}(X(x_1, \xi), X(x_2, \xi)) \]
Introduction
Probabilistic description of random quantities

- **Random variables**: $X(\xi) : \Omega \mapsto \mathbb{R}^d$, $d = 1, 2, 3$
  - essential Info: probability distribution

- **Random process**: temporal random distributed, $X(t, \xi) : T \times \Omega \mapsto \mathbb{R}^d$
  - essential Info: correlation function
    $$C(t_1, t_2) = \text{corr}(X(t_1, \xi), X(t_2, \xi))$$

- **Random field**: spatial random distributed, $X(x, \xi) : D \times \Omega \mapsto \mathbb{R}^d$
  - essential Info: correlation function
    $$C(x_1, x_2) = \text{corr}(X(x_1, \xi), X(x_2, \xi))$$

- **General random field**: Spatio-Temporal random distributed
  $X(x, t, \xi) : D \times T \times \Omega \mapsto \mathbb{R}^d$
  - essential Info: correlation function
    $$C(x_1, t_1, x_2, t_2) = \text{corr}(X(x_1, t_1, \xi), X(x_2, t_2, \xi))$$
Goal: estimating a parameter set: \( \mathcal{P} = (p_1, p_2, \ldots, p_q) \in \mathbb{R}^q \)
to relate the observed response data:
\( U_{ij} = U_i(t_j, p), \quad i = 1, \ldots, d, \quad j = 1, \ldots, m \)
to the predictor data \( \hat{U}_i(t_j, \hat{P}) \):
\[
\arg \min_{\hat{P}} \| L_2(\hat{P}) \| = \sum_{i=1}^{d} \sum_{j=1}^{m} w_{ij} \left( U_i(t_j, p) - \hat{U}_i(t_j, \hat{P}) \right)^2
\]  
(9)

\( w_{ij} \): weights, (e.g. \( = \frac{1}{\sigma_{ij}^2} \) if standard deviation of the data known)