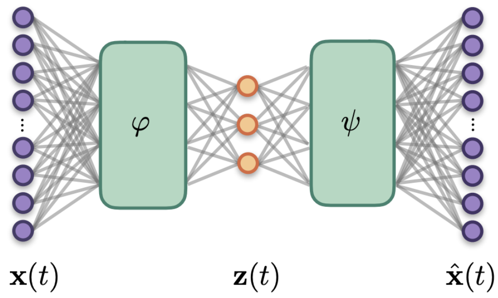
Lehrstuhl für Aerodynamik und Strömungsmechanik
Fakultät für Maschinenwesen
Technische Universität München

# Learning Fluid Mechanics from Data

**Motivation:**

The discovery of governing equations from data is of central importance in many physical and engineering applications. Often, one has access to measurements of a dynamical system but no knowledge of the underlying physical laws. This problem can be generally stated as $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mu)$, where $\mathbf{x}$ is the state vector of a dynamical system described by the unkown dynamics $\mathbf{f}$, $\dot{\mathbf{x}}$ is the time derivative, and $\mu$ represents a scalar parameter of the system. For example, the dynamics of the nonlinear pendulum are described by $\dot{x} = \sin(x)$, where $f(x) = \sin(x)$. Learning the governing equations, i.e. learning $\mathbf{f}$, from data, i.e. measurements of $\mathbf{x}$, improves the physical understanding of the dynamical system and potentially leads to improved model generation.

One possibility to approximate the unknown equations is the Dynamic Mode Decomposition (DMD) which assumes linear dynamics, $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$. Such a linear dynamical system can then be solved analytically via a harmonic ansatz which yields a decomposition of the dynamics into growing, decaying, and oscillating modes. A nonlinear generalization of DMD is given by Koopman analysis. Here, we seek to find a coordinate transformation, $\mathbf{z} = \varphi(\mathbf{x})$, that linearizes the dynamics. Such a coordinate transformation can be learned by an artificial neural network.




Schematic of a coordinate transformation via neural networks. Figure taken from Champion et al. (2019).

**Project Goal:**

The project seminar is divided into two parts. In the first part, students partake in lectures with hands-on exercise sessions. The second part of the seminar is the project work.

The theory part covers the following topics:

- Introduction to the Python programming language
- Discretizing and solving of Partial Differential Equations via the Finite Difference Method
- Linear models for regression
- Introduction to Deep Learning, i.e. Artificial Neural Networks in Tensorflow
- Dynamic Mode Decomposition and Koopman analysis
- Sparse Identification of Nonlinear Dynamics (SINDy)

In the group project, students will use aforementioned methods to identify the incompressible two-dimensional Navier-Stokes equations from data snapshots generated by numerical simulations.

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